Intro Video: Section 4.1
Maximum and Minimum Values

Math F251X: Calculus I

Some definitions

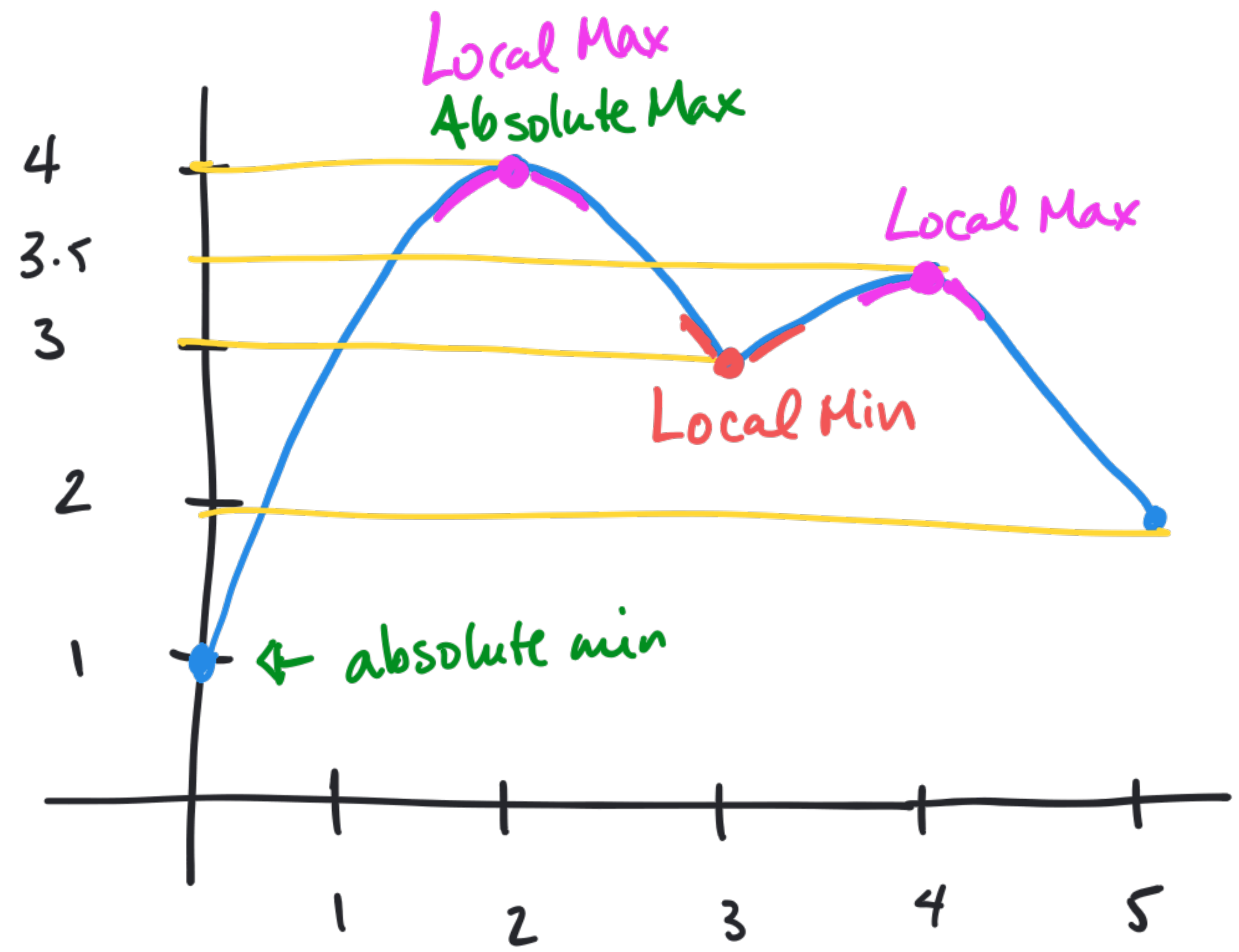
Setup: f is a function with domain D and $c \in D$.

- We say f has a absolute maximum at $x = c$ if $f(c) \geq f(x)$ for all $x \in D$.
- We say f has a absolute minimum at $x = c$ if $f(c) \leq f(x)$ for all $x \in D$
- We say f has a local maximum at $x = c$ if $f(c) \geq f(x)$ for all x "near" c
- We say f has a local minimum at $x = c$ if $f(c) \leq f(x)$ for all x "near" c

Note: "What is the absolute maximum of f"

$\longrightarrow$ this means we want a y-value/output

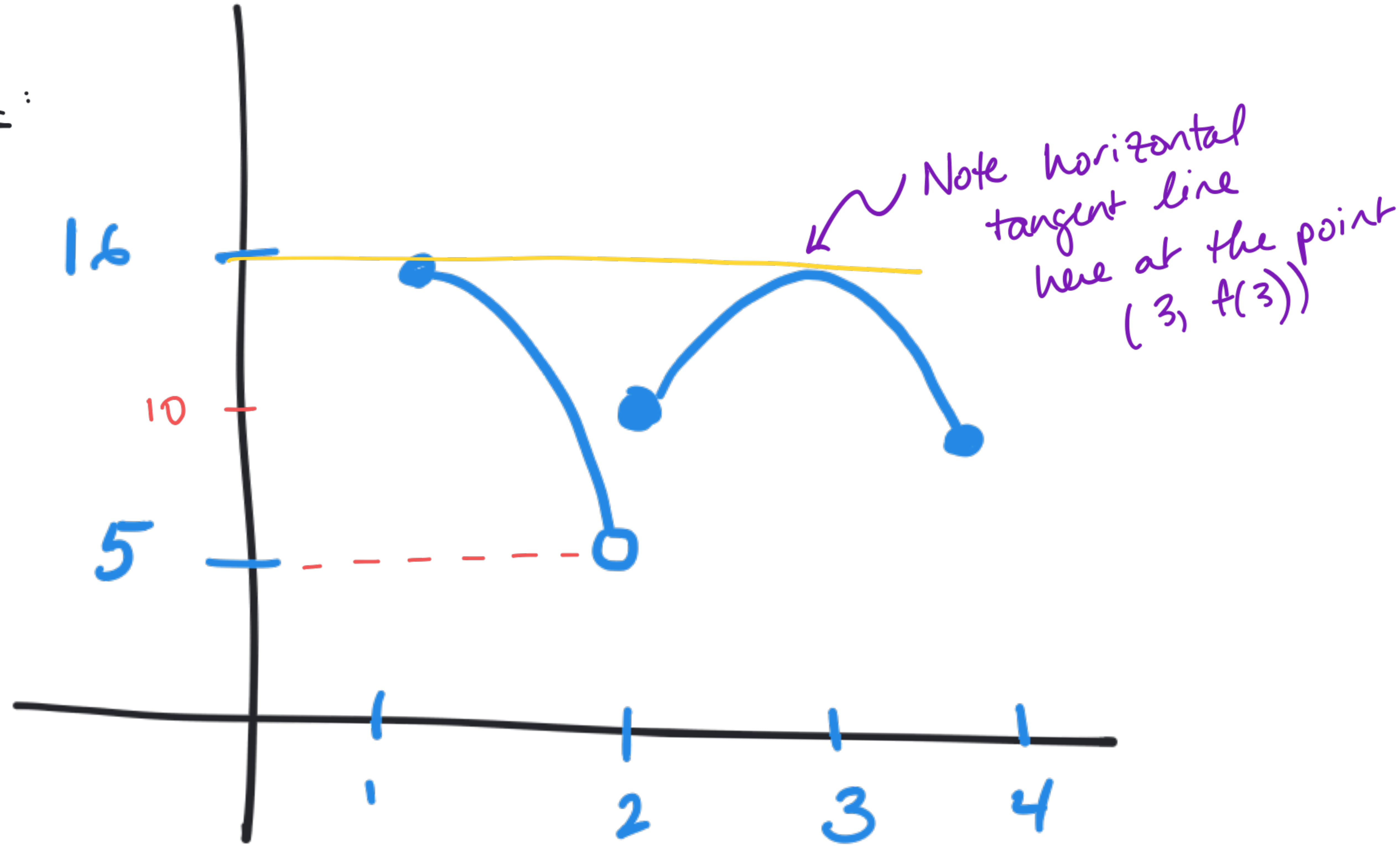
Example:



x	f(x)	feature of f
0	1	Absolute MIN
2	4	ABSOLUTE MAX (Also local max)
3	3	LOCAL MIN
4	3.5	LOCAL MAX
5	2	

The absolute maximum value of this function is $y = 4$ and it occurs at $x = 2$. The absolute minimum value of this function is $y = 1$, and it occurs at $x = 0$.

Example:



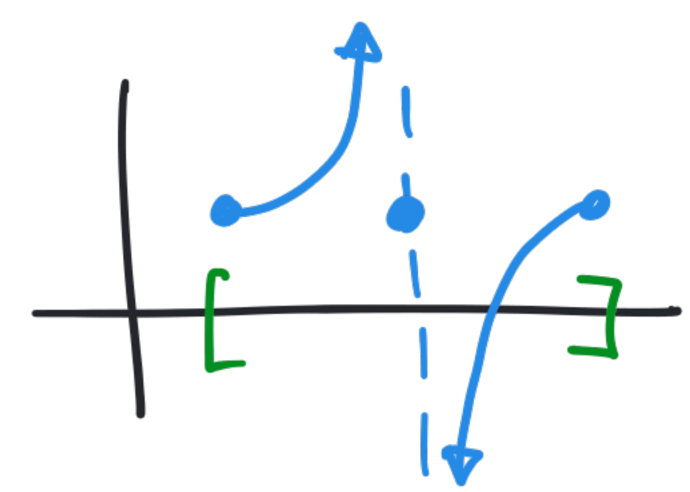
Domain: [1, 4] Absolute max = 16, which it reaches at x = 1 and x = 3. There is <u>no</u> absolute minimum.

Definition: A value $x = c$ is a critical point if $f'(c) = 0$ or $f'(c)$ is undefined.

Extreme Value Theorem: If f is continuous and f has a closed, bounded domain (that is, domain $= [a,b]$) then

① f has an absolute max value and an absolute min value

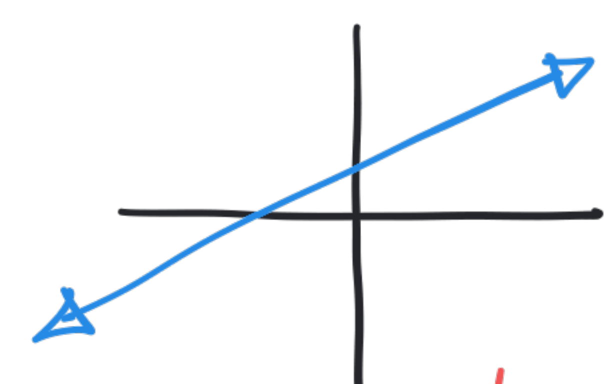
② the absolute max and min occur either at endpoints or at critical points



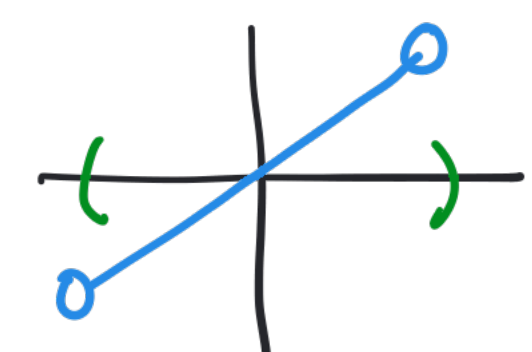
No abs max/min but closed, bounded domain

Recipe to find absolute max/min:

1) Find critical points
2) Evaluate f at critical points and end points
3) Identify absolute max/min



unbounded domain



Not closed

Example: Let $f(x) = 2x^3 - 3x^2 - 36x$. Find the absolute maximum and minimum values of $f(x)$ on $[-4, 6]$ and the x-values where they occur.

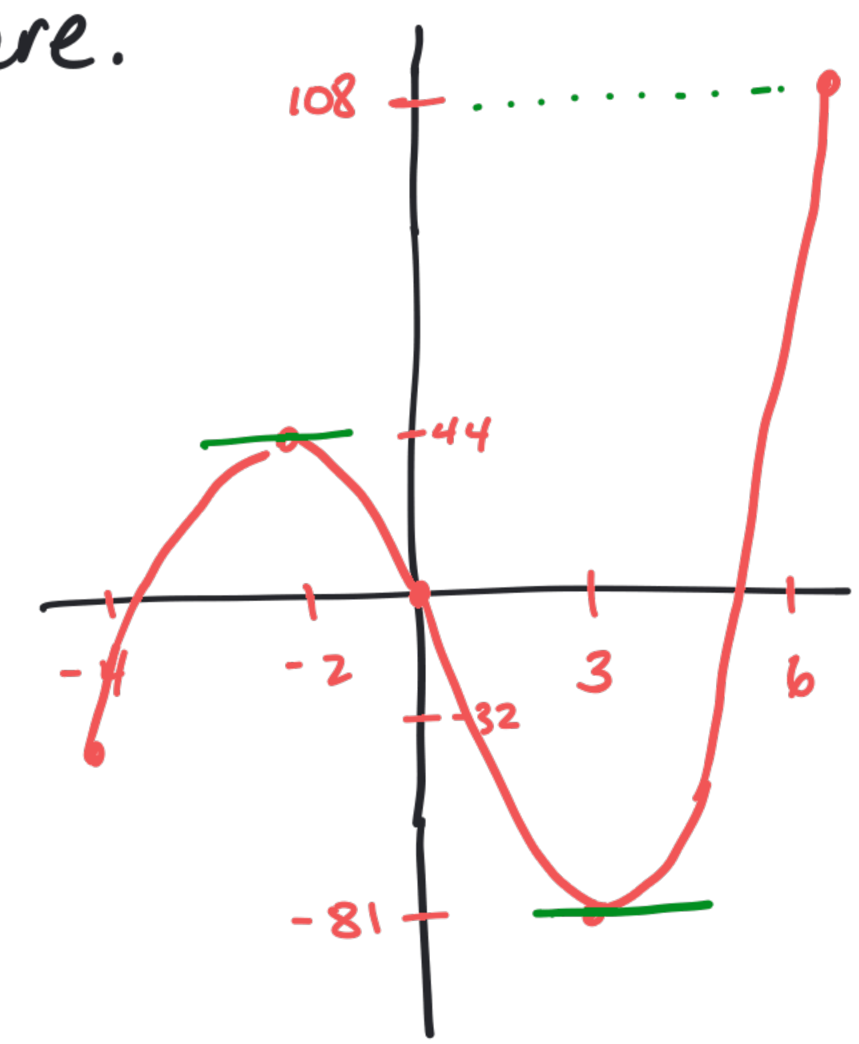
Find critical points: $f'(x) = 0 \Rightarrow 6x^2 - 6x - 36 = 0$

$\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3$ or $x = -2$.

Find where $f'(x)$ is undefined: nowhere.

x	$f(x)$	
-4	-32	
-2	44	
3	-81	← Absolute min $= -81$ at $x = 3$
6	108	← Absolute max $= 108$ at $x = 6$



Example: Find absolute max/min for $f(t) = \dfrac{\sqrt{t}}{1+t^2}$ on $[0,2]$.

Critical points: $f'(t) = \dfrac{(1+t^2)\left(\frac{1}{2}t^{-1/2}\right) - \sqrt{t}\,(2t)}{(1+t^2)^2}$

$$= \frac{1-3t^2}{2\sqrt{t}\,(1+t^2)^2}$$

$f'(t) = 0 \Rightarrow 1 - 3t^2 = 0$

$\Rightarrow 1 = 3t^2$

$\Rightarrow t = \frac{-1}{\sqrt{3}}$ or $\boxed{t = \frac{1}{\sqrt{3}}}$

$f'(t)$ is undefined $\Rightarrow$

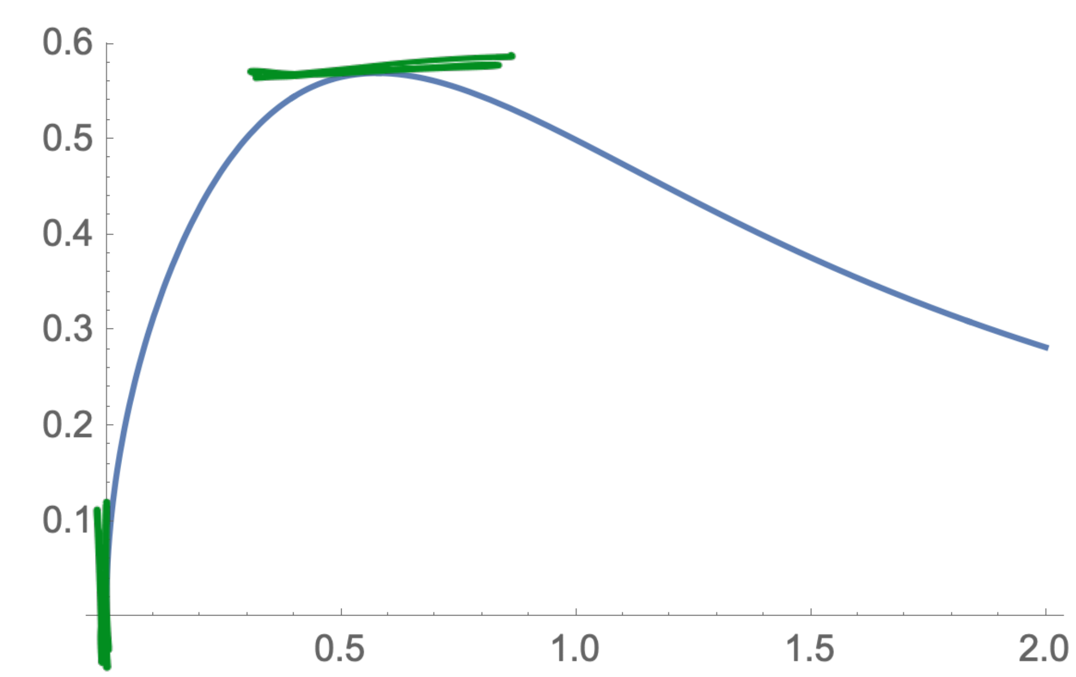
$2\sqrt{t}\,(1+t^2)^2 = 0 \Rightarrow$

$\boxed{t=0}$ or $1+t^2=0$ (Never!)

t	$f(t)$
0	0
$1/\sqrt{3}$	$\frac{3^{3/4}}{4} \approx 0.56$
2	$\frac{\sqrt{2}}{5} \approx 0.28$

$\leftarrow$ $y=0$ is absolute minimum at $x=0$

$\leftarrow$ $y = \frac{3^{3/4}}{4}$ is absolute max, at $x = 1/\sqrt{3}$



$1/\sqrt{3} \approx 0.557$